Nature Inspired Computing and Applications

Topics: Maxima, Minima, and Saddle Point for Single Variable and Multiple Variable Optimization Function, Realization

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Single Variable: Maxima, Minima and Saddle point

To find out maxima/ mimina/ saddle point for unconstraint & single variable:

Step1: Equate the gradient/ derivative of the function to zero to determine stationary point.

Step2: Maxima and minima are located at stationary point. Therefore, find out second derivative at stationary point.

Step3: If, IInd derivative

+ve	nature of extreme is minimum
-ve	nature of extreme is maxima
0	Saddle point, no minima or maxima

Single Variable: Maxima, Minima, & saddle point

$$f(x) = x^3 - 9x^2 + 24x \qquad \frac{df}{dx} = 3x^2 - 18x + 24 = x^2 - 6x + 8$$

Two stationary points, roots of $x^2 - 6x + 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4*1*8}}{2*1} => x = 2, x = 4$$

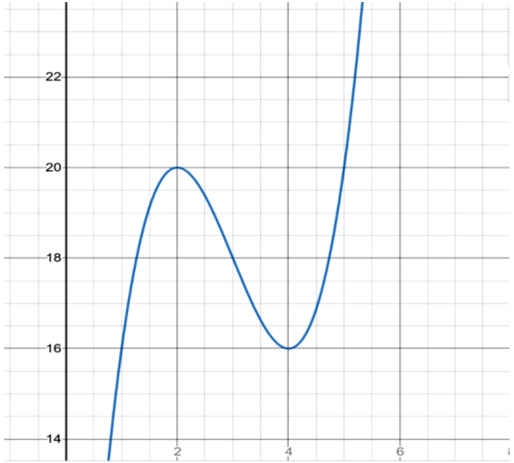
$$f'' = \frac{d^2 f}{dx^2} = 2x - 6$$

$$for\ x = 2, f''(2) = 2*2 - 6 = -2$$ **Maxima**

$$f(x) = 2^3 - 9*2^2 + 24*2 = 8 - 36 + 48 = 20$$

$$for\ x = 4, f''(4) = 2*4 - 6 = 2$$ **Minima**

$$f(x) = 4^3 - 9*4^2 + 24*4 = 64 - 144 + 96 = 16$$



Multiple Variable: Maxima, Minima and saddle point

To find out maxima/ mimina/ saddle point for unconstraint & single variable:

Step1: Equate the Jacobian derivative of the function to zero to determine stationary points.

Step2: Maxima and minima are located at stationary point. Therefore, find out Hassian at stationary point.

Step3: If, Hassian

definite +ve	nature of extreme is minimum
definite -ve	nature of extreme is maxima
definite 0	Saddle point, no minima or maxima

Multivariable: Maxima, Minima, & saddle point

$$f(x) = 3x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_1x_3 - 6x_1 - 4x_2 - 2x_3$$

$$J = \begin{vmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{vmatrix}$$

$$\frac{\partial f}{\partial x_1} = 6x_1 - 2x_2 + 2x_3 - 6$$

$$\frac{\partial f}{\partial x_2} = -2x_1 + 4x_2 - 2x_3 - 4$$

$$\frac{\partial f}{\partial x_3} = 2x_1 - 2x_2 + 2x_3 - 2$$

$$J = 0 \rightarrow \begin{vmatrix} 6 & -2 & 2 \\ -2 & 4 & -2 \\ 2 & -2 & 2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 6 \\ 4 \\ 2 \end{vmatrix}$$

Stationary Points

$$\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 1 \\ 3 \\ 3 \end{vmatrix}$$

$$H = \begin{vmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} & \cdots & \cdots & \frac{\partial^2 f}{\partial x_1 x_n} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \cdots & \frac{\partial^2 f}{\partial x_2 x_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 f}{\partial x_n x_1} & \frac{\partial^2 f}{\partial x_n x_2} & \cdots & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{vmatrix}$$

$$\frac{\partial^2 f}{\partial x_1^2} = 6, \quad \frac{\partial^2 f}{\partial x_1 x_2} = -2, \quad \frac{\partial^2 f}{\partial x_1 x_3} = 2$$

$$\frac{\partial^2 f}{\partial x_2 x_1} = -2, \quad \frac{\partial^2 f}{\partial x_2^2} = 4, \quad \frac{\partial^2 f}{\partial x_2 x_n} = -2$$

$$\frac{\partial^2 f}{\partial x_1 x_3} = 2, \quad \frac{\partial^2 f}{\partial x_2 x_3} = -2, \quad \frac{\partial^2 f}{\partial x_3^2} = 2$$

Hasian Matrix

$$H = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 4 & -2 \\ 2 & -2 & 2 \end{vmatrix}$$

Principal Determinant

$$H_1 = 6 > 0$$

$$H_2 = \begin{vmatrix} 6 & -2 \\ -2 & 4 \end{vmatrix} = 20 > 0$$

$$H_3 = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 4 & -2 \\ 2 & -2 & 2 \end{vmatrix} = 16 > 0$$

All +ve definite matrix, Hence stationary point corresponds to minima

Realization

- Two or more solution with same objective function. Different decision variable but same solution.

$$f(x_1 x_2) = \max(x_1 x_2)$$

$$s.t.\ \ x_1 + x_2 = 3$$

$$x_1 - x_2 \leq 1$$

$$x_1, x_2 \geq 0$$

	x_1	x_2	f
Sol1	1	2	2
Sol2	2	1	2

Problem Statement: Design a Cylindrical can using two parameters: diameter d and height h. Consider a case that the can needs to have a volume of at least 300ml and the objective of the design is to minimize the cost of can material.

Objective Function: $$Minimize\ \ f(\mathrm{d}, \mathrm{h}) = \mathrm{c}\left(\frac{\pi \mathrm{d}^2}{2} + \pi \mathrm{dh}\right)$$

$$Subject\ to\ g(d, h) = \frac{\pi d^2 h}{4} \geq 300$$