Understanding the Step by Step Procedure of PSO

(Note: In this write up all random numbers are generated using Matlab rand function)

Consider the optimization problem as follows:

$$Minimize\, f(x) = x_1^2 + x_2^2, \qquad -5 \le x_1, x_2 \le 5$$

First we generate swarm of size 5 randomly using uniform distribution in the range (-5, 5).

x =

2.7045	4.8030
4.5974	2.8793
1.8710	4.0528
1.6400	1.3202
3.3392	0.9963

Similarly the velocity vector is generated uniformly in the range (0, 1) (This may be [-7,7]).

V =

0.4752	0.6987
0.4141	0.4020
0.7797	0.9433
0.6183	0.4749
0.2530	0.9398

Let Vmax (maximum velocity bound) is 7, i.e., -7 <= V <=7.

Initial fitness vector is

30.3831
29.4265
19.9258
4.4325
12.1429

Minimum is 4.4325, i.e. x_4 is the best solution of this swarm. We call this as **gbest**. Also since no previous iteration exists so every particle's current position is also the pbest position.

Now we proceed to the next iteration using PSO update equations
(All calculations are carried out component wise)

For first particle

For first component

Velocity update:

$$v_{11} = v_{11} + c_1 r_1 (pbest_{11} - x_{11}) + c_2 r_2 (gebst_1 - x_{11})$$

v_{11} = 0.4752 + 2*0.34*(2.7045 - 2.7045) + 2*0.86*(1.6400 - 2.7045)
= - 1.35574

Position Update:

$$x_{11} = x_{11} + v_{11}$$

x_{11} = 2.7045+(-1.35574)
= 1.34876

Since 1.34876 lies in the range (-5, 5), so we accept this solution.

For second component

$$v_{12} = v_{12} + c_1 r_1 (pbest_{12} - x_{12}) + c_2 r_2 (gbest_2 - x_{12})$$

v_{12} = 0.6987+2*0.47*(4.8030-4.8030)+2* 0.91*(1.3202-4.8030)
= - 5.351696

$$x_{12} = x_{12} + v_{12}$$

x_{12} = 4.8030 - 5.351696 = - 0.5486

Again since - 0.836996 also lies in the range (-5, 5) so we accept it.
Thus the first particle after PSO update equations becomes:
x_1 = (1.34876, - 0.5486)

We update all the particles using same procedure.
Second Particle:
v_{21} = 0.4141+2*0.34*(4.5974 -4.5974)+2*0.86*(1.6400 -4.5974)
= - 4.672628
x_{21} = 4.5974 +(- 4.672628)
= - 0.075228
Again since - 0.075228 also lies in the range (-5, 5) so we accept it.

v_{22} = 0.4020 + 2*0.12*(2.8793-2.8793) + 2*0.06*(1.3202-2.8793)
= 0.214908
x_{22} = 2.8793 + (0.214908)
= 3.094208
Thus the second particle after PSO update equations becomes:
x_2 = (- 0.075228, 3.094208)

Third Particle:
v_{31} = 0.7797 + 2*0.98*(1.8710 -1.8710) + 2*0.86*(1.6400 -1.8710)
= 0.3824
x_{31} = 1.8710+(0.3824)
= 2.2534

v_{32} = 0.9433+2*0.69*(4.0528-4.0528)+ 2*0.34*(1.3202-4.0528)
= -0.9149
x_{32} = 4.0528+(-0.9149)
=3.1379
Thus the third particle after PSO update equations becomes:
x_3 = (2.2534, 3.1379)

Fourth Particle:
v_{41} = 0.6183 + 2*0.18*(1.6400 -1.6400) + 2*0.23*(1.6400 -1.6400)
= 0.6183
x_{41} = 1.6400+(0.6183)
= 2.2583

v_{42} = 0.4749+2*0.61*(1.3202-1.3202)+ 2*0.04*(1.3202-1.3202)
= 0.4749
x_{42} = 1.3202 + (0.4749)
= 1.7951
Thus the fourth particle after PSO update equations becomes:
x_4 = (2.2583, 1.7951)

Fifth Particle:

$v_{51} = 0.2530 + 2*0.09*(3.3392 - 3.3392) + 2*0.39*(1.6400 - 3.3392)$
$= -1.0724$

$x_{51} = 3.3392 + (-1.0724)$
$= 2.2668$

$v_{52} = 0.9398 + 2*0.65*(0.9963 - 0.9963) + 2*0.10*(1.3202 - 0.9963)$
$= 1.0046$

$x_{52} = 0.9963 + (1.0046)$
$= 2.0009$

Thus the fifth particle after PSO update equations becomes:
$x_5 = (2.2668, 2.0009)$

Finally the updated swarm is:

$x_1 = (1.3487, -0.5486)$
$x_2 = (-0.0752, 3.0942)$
$x_3 = (2.2534, 3.1379)$
$x_4 = (2.2583, 1.7951)$
$x_5 = (2.2668, 2.0009)$

and the fitness vector is:
2.1200
9.5797
14.9242
8.3223
9.1420

Clearly, it can be seen that the least fitness is 2.5194 which corresponds to the first particle. Therefore, gbest for updated swarm is x_1.

Now we compare this gbest with the previous gbest, obviously updated gbest is better so we replace old gbest with this new one. If updated gbest would not be better then old gbest is carry forward for the next iteration.

Now we see the update mechanism of pbest:

It should be noted that gbest is for whole swarm and pbest is for a particular particle.

For first particle:
Fitness in the previous swarm = 30.3831
Fitness in the current swarm = 2.5194
Clearly, the fitness of current swarm is better than that of its previous, so we set $pbest_1 = (1.3487, -0.5486)$.
If it would be the opposite case i.e. if the fitness of current swarm would not be better than that of its previous then the current pbest will be the old pbest.

Similarly,
For second particle: $pbest_2 = (-0.0752, 3.0942)$
For third particle: $pbest_3 = (2.2534, 3.1379)$
For fourth particle: $pbest_4 = (1.6400, 1.3202)$
For fifth particle: $pbest_5 = (2.2668, 2.0009)$
The same procedure is continued until the termination criterion is attained.